The figures show that the solid curves are in excellent agreement with the measurements. However, the losses of 0.5 dB and 0.7 dB occurring at resonance for $\theta = 75°$ in Figs. 6 and 7, respectively, are larger than those predicted by the theory (less than 0.1 dB). The difference is attributed to ohmic losses in the grids which were neglected in the theory.

ACKNOWLEDGMENT

The author wishes to thank F. A. Pelow for assistance in performing the experiment.

REFERENCES

[1] A. A. M. Saleh, "An adjustable quasi-optical bandpass filter—Part I: Theory and design formulas," this issue, pp. 728–734.
[2] N. Marcuvitz, *Waveguide Handbook* (Mass. Inst. Technol. Radiation Lab. Series), vol. 10. New York: McGraw-Hill, 1951, pp. 280–285.
[3] R. E. Collin, *Field Theory of Guided Waves*. New York: McGraw-Hill, 1966, pp. 338–348 and p. 366, Problem 8.10.
[4] ——, *Field Theory of Guided Waves*. New York: McGraw-Hill, 1966, p. 452, Problem 10.6.
[5] L. A. Weinstein, *The Theory of Diffraction and the Factorization Method: Generalized Wiener-Hopf Technique* (Electromagnetics Series), vol. 3. Boulder, Colo.: Golem, pp. 267–281.
[6] N. Marcuvitz, *Waveguide Handbook* (Mass. Inst. Technol. Radiation Lab. Series), vol. 10. New York: McGraw-Hill, pp. 285–289.
[7] T. Larsen, "A survey of the theory of wire grids," *IRE Trans. Microwave Theory Tech.*, vol. MTT-10, pp. 191–201, May 1962.
[8] G. G. MacFarlane, "Surface impedance of an infinite parallel-wire grid at oblique angles of incidence," *J. Inst. Elec. Eng.* vol. 93, pt. IIIA, pp. 1523–1527, 1946.
[9] J. R. Wait, "Reflection at arbitrary incidence from a parallel wire grid," *Appl. Sci. Res.*, vol. 4, sect. B, pp. 393–400, 1954.
[10] E. A. Lewis and J. P. Casey, Jr., "Electromagnetic reflection and transmission by gratings of resistive wires," *J. Appl. Phys.*, vol. 23, pp. 605–608, June 1952.
[11] J. K. Skwirzynski and J. C. Thackray, "Transmission of electromagnetic waves through wire gratings," *Marconi Rev.*, vol. XXII (Theory), pp. 77–90; (Experimental), pp. 91–98, Sec. Quart., 1959.
[12] V. Twersky, "On scattering of waves by the infinite grating of circular cylinders," *IRE Trans. Antennas Propagat.*, vol. AP-10, pp. 737–765, Nov. 1962.
[13] G. R. Bird and M. Parrish, Jr., "The wire grid as a near-infrared polarizer," *J. Opt. Soc. Amer.*, vol. 50, pp. 886–891, Sept. 1960.
[14] J. B. Young, H. A. Graham, and E. W. Peterson, "Wire grid infrared polarizer," *Appl. Opt.*, vol. 4, pp. 1023–1026, Aug. 1965.
[15] H. Hass and M. O'Hara, "Sheet infrared transmission polarizers," *Appl. Opt.*, vol. 4, pp. 1027–1031, Aug. 1965.
[16] J. P. Auton, "Infrared transmission polarizers by photolithography," *Appl. Opt.*, vol. 6, pp. 1023–1927, June 1967.
[17] P. K. Cheo and C. D. Bass, "Efficient wire-grid duplexer polarizer for CO_2 lasers," *Appl. Phys. Lett.*, vol. 18, pp. 565–567, June 1971.
[18] H. L. Garvin and J. E. Kiefer, "Wire-grid polarizers for 10.6-μm radiation," Dig. Tech. Papers, IEEE/OSA Conf. Laser Engineering and Applications, pp. 100–101, May–June 1973.
[19] N. Hill and S. Cornbleet, "Microwave transmission through a series of inclined gratings," *Proc. Inst. Elec. Eng.*, vol. 120, pp. 407–412, Apr. 1973.
[20] L. Weinberg, "Scattering matrices and matrices for multiterminal networks," in *Amplifier Handbook*, R. F. Shea, Ed. New York: McGraw-Hill, 1966, ch. 4, pp. 4-23–4-25.
[21] C. G. Montgomery, R. H. Dicke, and E. M. Purcell, *Principles of Microwave Circuits* (Mass. Inst. Technol. Radiation. Lab. Series), vol. 8. New York: McGraw-Hill, 1948, pp. 150–151.

Short Papers

Reduction of the Attenuation Constant of Microstrip

I. J. ALBREY AND M. W. GUNN

Abstract—**Modifications to a normal microstrip transmission line are proposed, with the aim of reducing the attenuation constant of the line. The results of a computer analysis of a structure containing multilayers of dielectric show that significant reductions in attenuation appear possible.**

INTRODUCTION

The advent of solid-state microwave devices has given great impetus to the development of microwave integrated circuits, which are based on a microstrip structure consisting of conducting strips separated from a ground plane by a substrate material with a high dielectric constant. Methods for the calculation of characteristic impedance, capacitance, and wavelength of such structures were established in the 1950's [1], but the first accurate calculations did not appear until the 1960's [2]. This two-conductor structure can be effectively shielded by enclosure in an appropriately large metal container, but as pointed out by Brenner [3], such transmission lines are beset with problems of inhomogeneity of the substrate, narrow strip widths for typical characteristic impedance levels (up to say 150 Ω), and high attenuation. The suspended substrate transmission line, which largely overcomes such difficulties, has thus become popular.

This short paper reports some results of an investigation into possible methods of reducing the attenuation of microstrip. It is proposed that the attenuation constant of a normal suspended substrate transmission line can be reduced by removing the substrate from the immediate vicinity of the conducting strip. The investigation was prompted by work on the use of loading in the form of a "shell" of dielectric to reduce the attenuation of a coaxial cable [4].

Such a modified structure would have the general form shown in Fig. 1 where dielectric ϵ_2^* is to be regarded as the substrate material that was originally in contact with the center conducting strip, but has now been removed a finite distance from the strip. The other regions consist of low-loss low dielectric constant material (ϵ_1^* and ϵ_4^* are air), and ϵ_3^* supports the center strip.

METHOD OF ANALYSIS

This multilayered structure has been analyzed on the assumption that a TEM field pattern exists. For this to be valid, the wavelength must be much greater than the transverse dimensions of the line, and so a frequency of 1 GHz (λ_0 equals wavelength in air-filled line = 30 cm) is chosen. Several methods of analysis are available, the variational approach [5] being chosen because of its direct nature and its accuracy, particularly in the calculation of attenuation. The method is based on a variational technique using Green's functions and considers the center conductor to be infinitely thin.

Manuscript received November 27, 1973; revised January 24, 1974. This work was supported in part by the Australian Research Grants Committee.
The authors are with the Department of Electrical Engineering, University of Queensland, St. Lucia, Queensland, Brisbane, Australia.

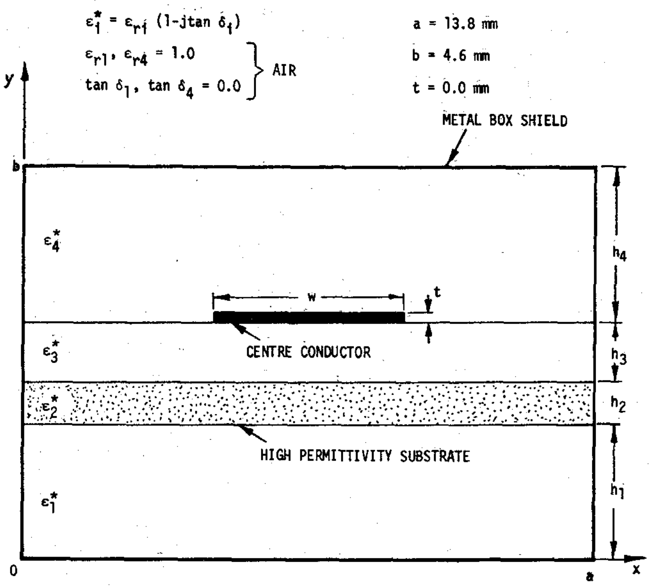

Fig. 1. Strip transmission line containing four dielectric layers.

TABLE I

ATTENUATION CONSTANTS FOR MICROSTRIP LINE OF FIG. 1[a]

ϵ_{r2}	w (mm)	λ/λ_0	α N/m	α dB/λ
6.10	0.743	0.480	0.114	0.143
9.35	0.496	0.400	0.152	0.159
11.00	0.422	0.363	0.175	0.166

[a] $h_1 = 0 = h_3$, $h_2 = 0.5$ mm, and $Z_0 = 50\ \Omega$.

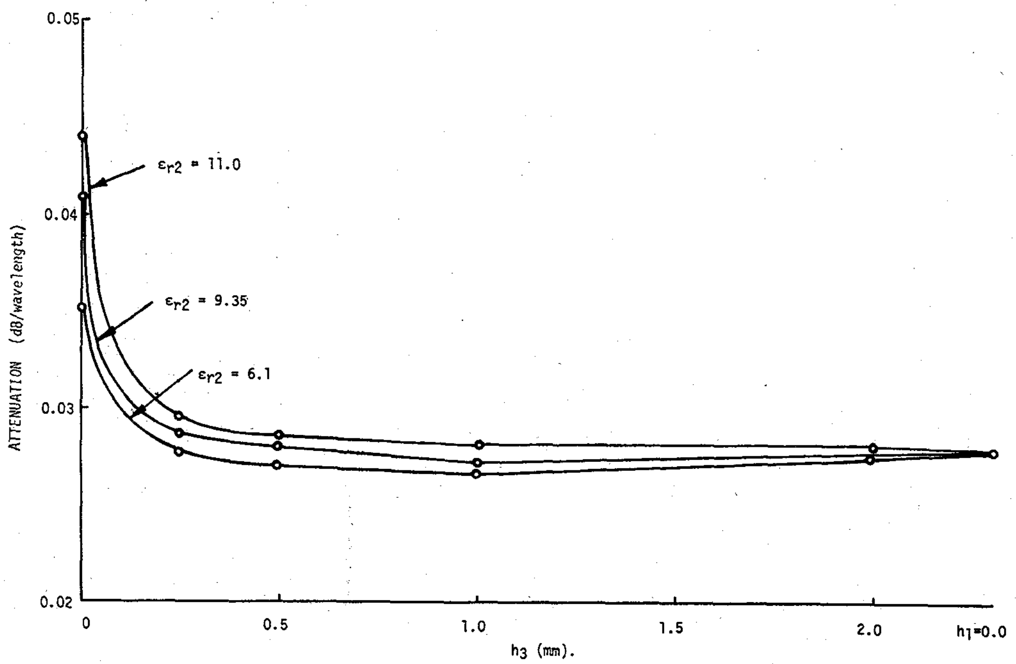

Fig. 2. Attenuation versus position of substrate for a fixed substrate thickness of the structure of Fig. 1 ($h_2 = 0.5$ mm, $h_4 = 1.84$ mm, $\epsilon_{r3} = 1.0$, $\tan \delta_3 = 0.0$, $\tan \delta_2 = 2 \times 10^{-4}$, $Z_0 = 50\ \Omega$).

Such an approach requires an estimate of the functional form of the charge distribution on the strip, this being taken in the form of a cubic function of x (Fig. 1) in the present analysis. Alternatively, one could use the method of images [6], but it was considered that this method would result in equations that would be difficult to handle for this particular situation. The relaxation technique [7] could be used, but this does not appear to be sufficiently accurate for attenuation calculations. Also, the finite-difference method used by Brenner [3] may suffer the same disadvantage.

Referring again to Fig. 1, the following parameters are used in this investigation.

Substrate: $\epsilon_{r2} = 6.1$ or 9.35 or 11.0, $\tan \delta_2 = 2 \times 10^{-4}$.

Support: $\epsilon_{r3} = 1.0$, $\tan \delta_3 = 0.0$ (air); or $\epsilon_{r3} = 1.03$, $\tan \delta_3 = 3 \times 10^{-5}$ (polystyrene foam).

Conductors: Conductivity equals 5.8×10^7 S/m. The center strip is infinitely thin.

In order to provide a basis for comparison, it is appropriate that the attenuation for normal shielded microstrip be calculated, that is, the strip is on top of the high dielectric constant substrate, which, in turn, is on the bottom of the box ($h_1 = 0 = h_3$). In practice, substrate thicknesses are available in only a few fixed values, which typically lie in the range 0.25–0.625 mm (0.010–0.025 in). For our purposes, the substrate thickness (h_2) was chosen as 0.5 mm (0.020 in) and the strip width (w) was computed to give the characteristic impedance $Z_0 = 50\ \Omega$ for each of the three values of ϵ_{r2}. The attenuation of the lines may then be calculated, and the results are shown in Table I.

Consider now the modified suspended substrate line. A reasonable value for h_4 is about $0.4b$ [3], where b is the height of the box, that is, $h_4 = 1.84$ mm in this case. For a given position of the substrate, that is, for a given value of h_3, we may select h_2 and calculate w to give $Z_0 = 50\ \Omega$. In either case, once h_2 is determined, h_1 is also known. The limit of this procedure obviously occurs when h_1 becomes 0.

For the case of a fixed substrate thickness of $h_2 = 0.5$ mm and varying w as required to give $Z_0 = 50\ \Omega$, the attenuation for several values of h_3 for each of the three values of ϵ_{r2} was calculated. The results are shown in Fig. 2. Of particular interest here is a comparison between the attenuation of the conventional microstrip line (Table

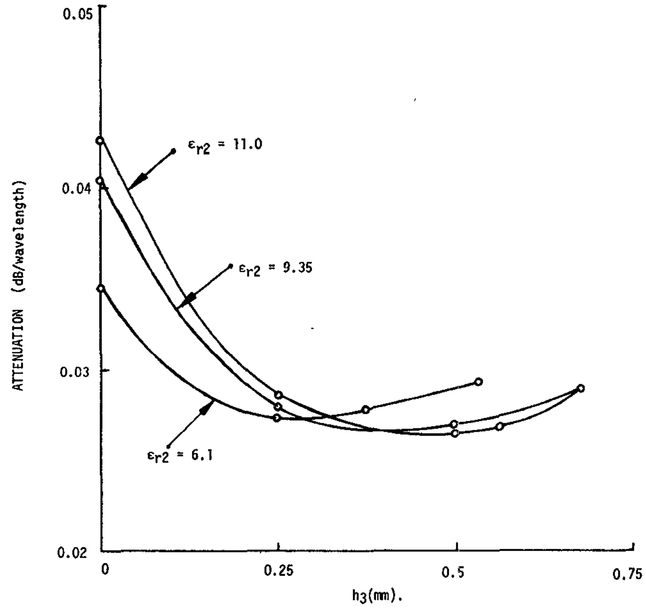

Fig. 3. Attenuation versus position of substrate for a variable substrate thickness of the structure of Fig. 2 (w = 4.6 mm, h_4 = 1.84 mm, ϵ_{r3} = 1.0, tan δ_3 = 0.0, tan δ_2 = 2.10^{-4}, Z_0 = 50 Ω).

I) and the suspended substrate line ($h_3 = 0.0$ in Fig. 2) for the same values of substrate thickness (0.5 mm). For $\epsilon_{r2} = 9.35$, we find an attenuation of 0.159 dB/λ in Table I as compared with the corresponding figure of 0.041 dB/λ in Fig. 2. Clearly, a large reduction (approximately 75 percent) in attenuation is achieved by using the suspended substrate configuration. Fig. 2 shows that this can be further improved upon by removing the substrate from the immediate vicinity of the center conductor.

For the alternative approach, w was kept fixed at 4.6 mm and h_2 varied as required to give $Z_0 = 50\ \Omega$. This procedure was carried out for several values of h_3 and the results are shown in Fig. 3. It should be noted that the results presented in Figs. 2 and 3 are for air as the support for the strip, which is obviously a physically unreal situation. However, calculations with polystyrene foam as the support showed that the attenuation of the structure was increased by less than 2 percent of the values shown in Figs. 2 and 3.

DISCUSSION

Comparison of Table I and Fig. 2 shows the significant reduction in attenuation obtained by suspending the substrate. In absolute attenuation figures with a substrate dielectric constant $\epsilon_{r2} = 11.0$, the attenuation of the normal microstrip structure is 0.175 N/m compared with the suspended substrate figure of 0.0212 N/m. With the proposed structure, the attenuation has been lowered further by removing the substrate from the immediate vicinity of the center conductor. As h_3 is increased, the attenuation passes through a minimum, as shown in Fig. 3. A more detailed plot of the relevant region in Fig. 2 would reveal a similar behavior.

In order to study the effects of moving the substrate on the losses in the various sections of the structure in Fig. 1, it is necessary that the total power transmitted be the same in all cases. Because every line studied has the same characteristic impedance, this means that the current in every line remains the same. This, in turn, requires the evaluation of a constant multiplier in each case for the functional form of the charge distribution used in the analysis. The value of this multiplier varies from structure to structure and depends on the product of the total charge on the strip and the phase velocity.

Without entering into great detail, this investigation has revealed that the major cause of loss in the line is the interface between the center conductor and the substrate. This loss typically accounts for 80 percent of the total loss and is reduced by up to 60 percent for the cases studied with only a small movement of the substrate, thereafter decreasing at a very much lower rate as h_3 increases. As the substrate is moved further from the strip, its influence on the fields around the strip decreases, resulting in a small increase in the losses in the top of the strip and the top of the box. Also, there is an increase in the losses in the bottom of the box as the substrate approaches that surface. The overall result is a small increase in losses as h_1 approaches 0, which results in a small increase in attenuation of the line. It should be noted that the losses in the sidewalls and the dielectric losses account for only a small fraction of the total and simply decrease for increasing h_3. They are therefore not of any great significance in this discussion. Calculation of the attenuation constant from the expression $\alpha = P_L/2P_T$, where P_L is the average power loss per unit length and P_T the average power transfer in the direction of propagation at any point, thus shows a reduced attenuation figure for the line.

A similar behavior occurs in the transmission lines used for the results of Fig. 3. As has been pointed out, these lines have a fixed value for w of 4.6 mm, which is smaller than any of the strip widths of the transmission lines used for the results of Fig. 2. The minimum attenuation obtained by this procedure for a given ϵ_{r2} is greater than that found in the former approach. This is due primarily to the fact that before the losses in the bottom of the strip have been reduced by as great a proportion as in the first method, the bottom surface of the substrate is in close proximity to the bottom of the box with consequent increasing losses in this conductor surface. Also due to the thicker substrate, the field strength in the upper section of the line is reduced with consequent reduced losses in the top of the strip and top of the box. Again, the dielectric losses and losses in the sidewalls are only a small fraction of the total. The overall result is that, for given values of ϵ_{r2} and h_3, a smaller total loss and hence a reduced attenuation is obtained.

Referring to the first method of approach, two other substrate thicknesses were chosen, and the attenuation figures for each thickness–permittivity combination are shown in Table II. These figures are for the conventional suspended substrate structure.

TABLE II
ATTENUATION CONSTANTS FOR MICROSTRIP LINE OF FIG. 1[a]

ϵ_{r2}	h_2 (mm)	λ/λ_0	α N/m	α dB/λ
6.1	0.25	0.920	0.0134	0.0321
	0.125	0.953	0.0121	0.0302
9.35	0.25	0.880	0.0154	0.0354
	0.125	—	—	—
11.0	0.25	0.873	0.0165	0.0376
	0.125	0.928	0.0136	0.0330

[a] h_3 = 0, h_4 = 1.84 mm, and Z_0 = 50 Ω.

It is clearly seen that the attenuation decreases with decreasing permittivity and thickness of the substrate. In the limiting case where the line is completely air filled, the attenuation is 0.028 dB/λ (0.0107 N/m). For each of these two new thicknesses, the same procedure of moving the substrate and adjusting the strip width was carried out. As in the previous case, a reduced attenuation was obtained. However, the minimum attenuation found for any given value of substrate thickness and permittivity was approximately 0.028 dB/λ (0.0107 N/m), that is, the attenuation of an air-filled line.

CONCLUSION

It has been shown that the attenuation per wavelength of a suspended substrate microstrip transmission line is about one-quarter that of normal microstrip line and that this attenuation can be further reduced by removing the substrate from the immediate vicinity of the center strip. A reduction factor of one-half is readily obtainable. It has further been shown that the attenuation of the proposed multilayer structure can be made to be very close to that of an air-filled line which, in turn, appears to have the lowest possible attenuation.

Clearly, the results presented here are limited and the conclusions drawn from them depend on the validity of the theoretical assumptions [5]. Of particular concern is the assumed charge distribution on the center strip. All that is known about this is that it is much larger at the edges of the strip than at the center. This type of behavior has been observed experimentally by Dukes [8], and hence a cubic function seems a reasonable assumption. Obviously, there is no guarantee that such a representation is correct for all cases studied here, but a large amount of computation would be required in order to determine the best functional representation for every structure. However, it appears that the proposals put forward here could lead to a new form of shielded suspended substrate microstrip line. For situations where several circuits are etched upon the one substrate, high-permittivity substrate materials are required to isolate these circuits, and the multilayer structure proposed provides a means of reducing the losses introduced by such materials. Further work is in hand to define the effects on the electric field distribution of moving the substrate and to find an optimum structure.

REFERENCES

[1] *IRE Trans. Microwave Theory Tech.* (*Special Issue: Symposium on Microwave Strip Circuits*), vol. MTT-3, pp. 1–182, Mar. 1955.
[2] R. A. Pucel, D. J. Massé, and C. P. Hartwig, "Losses in microstrip," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 342–350, June 1968.
[3] H. E. Brenner, "Use a computer to design suspended-substrate IC's," *Microwaves*, vol. 7, pp. 38–45, Sept. 1968.
[4] I. J. Albrey and M. W. Gunn, "Attenuation reduction in coaxial cables," *Proc. Inst. Elec. Eng.*, vol. 118, pp. 1749–1751, Dec. 1971.
[5] E. Yamashita and K. Atsuki, "Strip line with rectangular outer conductor and three dielectric layers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 238–244, May 1970.
[6] P. Silvester, "TEM wave properties of microstrip transmission lines," *Proc. Inst. Elec. Eng.*, vol. 155, pp. 43–48, Jan. 1968.
[7] H. E. Stinehelfer, Sr., "An accurate calculation of uniform microstrip transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 439–444, July 1968.
[8] J. M. C. Dukes, "An investigation into some fundamental properties of strip transmission lines with the aid of an electrolytic tank," *Proc. Inst. Elec. Eng.*, vol. 103, sect. B, pp. 319–333, May 1956.

Waves Guided Between Open Parallel Concave Reflectors

F. J. TISCHER, FELLOW, IEEE, AND J. R. POTUKUCHI

***Abstract*—Wave propagation between two open parallel concave reflectors is usually considered by a wave-beam or a multiple-reflection approach. It is shown that the field distribution in elliptic waveguides, for specific wave modes, approaches that in open reflector waveguides.**

INTRODUCTION

A structure composed of two parallel cylindrical reflectors, as illustrated in Fig. 1, has attractive characteristics as a waveguide, particularly in the millimeter-wave region. Three methods for the analysis of such reflector waveguides are described in the literature. One is based on a description of the field distribution by transverse wave beams [1]–[3]; another one [4], [5] uses Huygen's principle applied to the iterative radiation from and reflection by the reflector surfaces. The third method [6], [7] analyzes the case as a boundary value problem by taking into consideration diffraction at the openings. In this analysis, the wave equation is being transformed into a parabolic partial differential equation. The use of these reported methods involves restrictions such as the limitation to confocal reflectors [1]–[6] and restrictions to specific ratios of cross-sectional dimensions associated with the choice of elliptic coordinates [6].

In the present short paper, it is shown that the wave propagation in open waveguides with concave reflectors also can be described by that in a waveguide with elliptic cross section. The analysis indicates that for large and transverse wavenumbers the field distribution in the closed elliptical waveguide approaches that of the open reflector-type waveguide. In contrast to the previously reported approaches, the new method does not require implementation of any of the above-mentioned restrictions.[1]

Manuscript received November 19, 1973; revised January 21, 1974.
The authors are with the Department of Electrical Engineering, North Carolina State University, Raleigh, N. C. 27607.

[1] A similar approach has been followed by Toraldo di Francia [11] in the analysis of a "flat-roof" resonator. This has been brought to the attention of the authors by one of the reviewers.

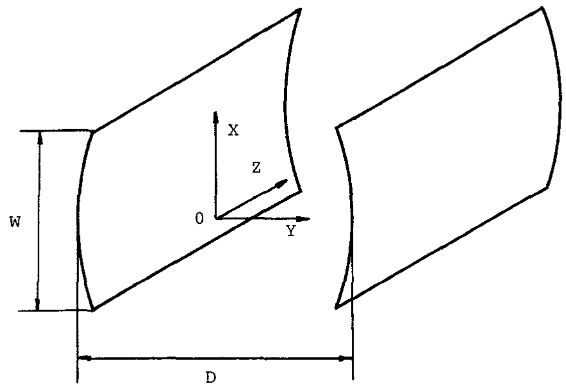

Fig. 1. Reflector guide.

BASIC RELATIONSHIPS

In this section the basic relationships for the field distribution in the elliptic waveguide are outlined. For large values of the parameter q, the equations then also describe the field distributions in the open waveguide with concave reflectors.

The transmission characteristics of elliptic waveguides have been studied by several investigators [8], [9]. An elliptic coordinate system is usually used in the analysis, as indicated in Fig. 2. The distributions of the various field components are found by solving the transverse wave equation for the longitudinal field components E_z and H_z in this coordinate system. Assuming harmonic time variations and wave propagation in the positive z direction (along the cylinder axis), the wave equations have the basic form

$$\frac{\partial^2\psi}{\partial\xi^2} + \frac{\partial^2\psi}{\partial\eta^2} + 2q(\cosh 2\xi - \cos 2\eta)\psi = 0 \qquad (1)$$

where $\psi = E_z$ for TM modes and $\psi = H_z$ for TE modes. Other parameters are

q $= k_c^2c^2/4$;
k_c^2 $= \omega^2\epsilon_0\mu_0 - k_z^2$;
$2c$ focal distance;
k_z propagation constant in z direction;
k_c propagation constant in transverse direction (also propagation constant for cutoff).

The basic solution of (1) is found by separation of variables and becomes, in customary writing,

$$\psi_{m,n} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{m,n}Ce_m(q_{m,n}\xi)ce_m(q_{m,n}\eta) \qquad (2a)$$

for even-field modes, and

$$\psi_{m,n} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{m,n}Se_m(q_{m,n}\xi)se_m(q_{m,n}\eta) \qquad (2b)$$

for odd-field modes.

The functions ce_m and se_m are the even and odd Mathieu functions of order m, and Ce_m and Se_m are the corresponding modified Mathieu functions. The order number m indicates the number of zeros of the Mathieu functions between $\eta = 0$ and $n = \pi/2$. In the present case, the functions with the order numbers m and n then represent the distributions of E_z and H_z, respectively, for the various wave modes. Evaluation of the Mathieu functions shows that for large values of q and k_c the functions Se_m and Ce_m are highly quasi-periodic. The number of zeros increases with increasing values of q. The functions, in the present case, represent the standing waves resulting from reflections between the upper and lower parts of the elliptic waveguide, shown in Fig. 2, along the Y axis. Considering the field distribution in direction of η (X direction), it is described by ce_m and se_m. For the fundamental mode designated by $m = 0$, the function ce_0 has no periodicities and zeros, and it increases monotonically when η varies between 0 and $\pi/2$. Typical examples of these functions are illustrated in Fig. 3 for various values of the parameter q. The curves indicate that for large values of q the magnitude of ce_0 is large in the vicinity of $\eta = \pi/2$ and becomes negligibly small near $\eta = 0$. This means that the field distributions